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# Efficient system for queues: a case study on a fast-food restaurant in Pahang

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**ABSTRACT**

In a fast-food restaurant, waiting for service is a common experience for customers. This study deals with *M/M/1* and *M/M/2/∞/FCFS*queues in a fast-food restaurant. The data collected underwent Chi-square test for Poisson and exponential distributions in the queuing model. We measured the performance of existing and proposed systems to compare which system is best to be applied at the fast-food restaurant. Measurements of the system's operation of the fast-food restaurant are analysed based on $L_q$, $W_s$, and $W_q$ of each model. In this study, we proved that multi-server, single-line model is better than multi-server, multi-line model and we discuss the relation between these two models.

*Keywords: Queuing Theory; Queuing Model; Fast-Food Restaurant; Queue Management; Poisson Distribution; Chi-Square Goodness Of Fit Test.*



## 1. INTRODUCTION

Queuing systems exist everywhere. We have all experienced waiting in line somewhere in our daily life; waiting for response in front of a computer, waiting for a bus inside campus, waiting on the phone to get technical help, waiting for services at a bank, *etc*. The time spent in waiting is most likely unwarranted. The analysis of the systems involved in queues or waiting lines is one of the best ways to assess efficiency. The efficiencies of queuing systems are determined by how much waiting takes place[1].

A queuing system is a model with the structure of people or customers arriving and joining queues to wait for services. After customers get the service, they will exit the system. Queuing theory analyses every aspect of waiting lines, including the arrival process, the service process, the queue discipline, the number of servers, and the number of customers in the queue. Queuing theory is used to develop efficient queues which will reduce customers waiting times, increase the number of customers that can be served in a system and optimize the operation cost of businesses [2].

## 1.2 PROBLEM FORMULATION

Fast-food restaurants are good examples of where problems of unwarranted waiting time occur especially during their peak hours. This analysis can be used as a performance measure and for better resource planning to fulfil the customers' satisfaction, to avoid loss of customers due to long waits and to optimize the total business operation costs.

This paper focuses on the application of queuing theory in waiting lines at a fast-food restaurant as an effort to increase its efficiency in the form of reduced waiting time. Only two types of systems are considered which are the multi-server, multi-line, single-phase system and the multi-server, single-line, single-phase system. The aim of this research to propose the better system that the restaurant should apply in order to be more efficient in terms of reduced customers' waiting time

## 2. THE CHI-SQUARE GOODNESS OF FIT TEST FOR POISSON DISTRIBUTION

Data is obtained from a popular fast-food restaurant in Pahang. The data were collected at the restaurant's peak hours; lunch time from 1200hrs to 1400hrs over a three-day period. Data collected are the time of arrival of each customer, waiting time, and service time. The Chi-Square Goodness of Fit Test is done to determine the distribution suitable for the data. Excel 2013 and QM for Windows V5 were used to analyse data and the queuing system respectively.

### 2.1 Expected Frequency

The restaurant is observed to adopt the multi-server, multi-line, single phase system with two servers; *2*M/M/1*. To start the analysis, data from *Day-1* is tabulated in Table 1 to determine the expected frequencies.

**Table 1** Frequencies of customer arrivals for data collected on *Day-1*

| Arrival, $n$ | Observed Frequency, $f_0$ | $nf_0$ |
|---|---|---|
| 0 | 8 | 0 |
| 1 | 14 | 14 |
| 2 | 14 | 28 |
| 3 | 8 | 24 |
| 4 | 5 | 20 |
| 5 | 1 | 5 |
| TOTAL | 50 | 91 |

From Table 1, the value of $\lambda$, the mean of a Poisson distribution is determined from the observed data by computing the mean of the data, where $\lambda = \frac{91}{50} = 1.82$. The probability of arrival with $\lambda = 1.82$ is then given by

$$P(x) = \frac{e^{-\lambda}\lambda^n}{n!}$$

The value of expected frequency for each arrival is shown in Table 2. Expected frequency is determined using the following formula

$$f_e = \left(\sum f_0\right)P(x) = 50P(x)$$

**Table 2** Observed and expected frequencies for data collected on *Day-1*

| Arrivals | Observed Frequency, $f_0$ | Probability $P(x)$ | Expected Frequency, $f_e$ |
|---|---|---|---|
| 0 | 8 | 0.1620 | 8.1013 |
| 1 | 14 | 0.2949 | 14.7443 |
| 2 | 14 | 0.2684 | 13.4174 |
| 3 | 8 | 0.1628 | 8.1399 |
| 4 | 5 | 0.0741 | 3.7036 |
| 5 | 1 | 0.0270 | 1.3481 |

Chi-square is sensitive to small expected frequencies, $f_e$ in the cells, thus we have to be careful in interpreting chi-square if one or more cells are less than 5 [3]. Whenever the expected frequency of a cell does not exceed 5, this cell needs to be combined with other cells until the above condition is satisfied. From Table 2 we noticed that the observed frequencies belonging to 4 and 5 arrivals are less than 5, therefore the observed frequency belonging to 4 and 5 arrivals are pooled up to make the observed frequency greater than 5 as shown in Table 3.

**Table 3** Observed and expected frequencies with $f_e > 5$ for data collected on *Day-1*

| Arrivals | Observed Frequency, $f_0$ | Expected Frequency, $f_e$ | $\frac{(f_0 - f_e)^2}{f_e}$ |
|---|---|---|---|
| 0 | 8 | 8.1013 | 0.0013 |
| 1 | 14 | 14.7443 | 0.0376 |
| 2 | 14 | 13.4174 | 0.0253 |
| 3 | 8 | 8.1399 | 0.0024 |
| 4 or 5 | 6 | 5.0518 | 0.1780 |

Lastly, we can find the value of Chi-square from the formula below:

$$\chi_{cal}^2 = \sum \frac{(f_0 - f_e)^2}{f_e} = 0.2445$$

Next, the critical value is determined by using function in Excel 2013. For this $\chi^2$test, the degrees of freedom is the number of categories minus one, minus the number of parameters. The degrees of freedom are $5 - 1 - 1 = 3$ for $\alpha = 0.05$. This function is used to compare observed results with expected ones in order to decide whether the original hypothesis is valid. Here the value is$\chi^2 = 7.8147$.

Clearly $\chi_{\mathrm{cal}}^2 < \chi_{0.05,4}^2 = 7.8147$. Hence the null hypothesis is accepted and thus there is no reason on the basis of this test for doubting that queuing model can be applied to this data. This also implies that the arrival pattern per 5-minute follows a Poisson distribution as presented in Figure 1.

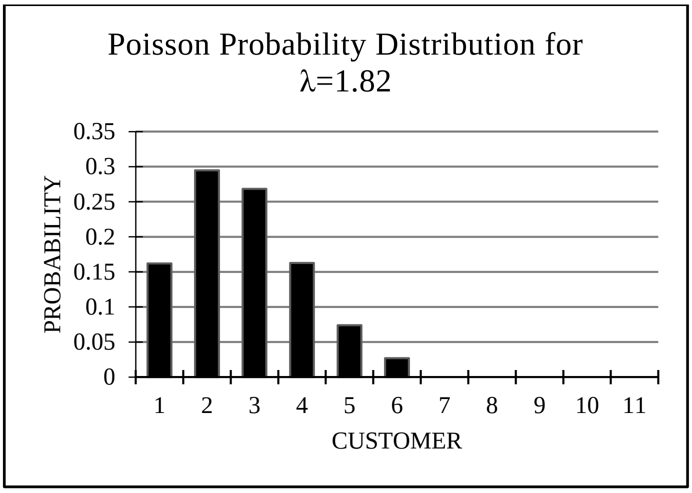


**Figure 1** Graph of Poisson probability distribution for λ=1.82

The analyses were repeated on *Day-2* and *Day-3* collected data. Data from these two days also follow the same pattern which lead us to conclude that the data follows the Poisson distribution and hence Queuing Theory analysis can be applied on these set of data.

## 3. EXISTING MODEL

The model that best described the queuing system in this fast-food restaurant is two servers with two queues; *2*M/M/1*. For this queuing system, it is assumed that arrivals follow a Poisson distribution and service times are distributed exponentially [4]. It is further assumed that only one queue is formed for each server. Therefore, each of these 2 servers are computed using *M/M/1* queues. We have

$$U = \rho = \frac{\lambda}{\mu} < 1$$

The probability $P_n$ of $n$ customers in the queuing system is given by

$$P_n = \rho^n P_0 P_{n+1} = \rho P_n$$

where,

$$P_0 = 1 - \rho = 1 - \frac{\lambda}{\mu}$$

The expected number of customers waiting in the queue is

$$L_q = \lambda W_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{1 - \rho}$$

The expected number of customers in the system is

$$L_s = \lambda W = L_q + \rho = \frac{\rho}{1 - \rho} = \frac{\lambda}{\mu - \lambda}$$

where the expected waiting time of a customer in the queue is

$$W_q = W - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L_q}{\lambda} = \frac{\rho}{\mu(1-\rho)}$$

and the expected waiting time that a customer spends in the system is

$$W_s = \frac{1}{\mu-\lambda} = W_q + \frac{1}{\mu} = \frac{1}{\mu(1-\rho)}.$$

## 4. PROPOSED MODEL

The system that we propose to replace the existing one is *M/M/2/∞/FCFS.* In this model customers' arrivals are described by Poisson distribution, come from infinite population, service time follows exponential distribution, with single queue, two servers [5]. This model is proposed because it is observed that it is fairer to the customers on the basis of First-Come-First-Served ideals. With the current model, there are cases where customers who arrive earlier leave later depending on the service rate of individual servers.We have

$$U = \frac{\rho}{c} = \frac{\lambda}{c\mu} < 1$$

The probability that there are no customers in the system is

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

where $c$ is number of servers. The probability $P_n$ of $n$ customers in the queuing system is given by

$$P_n = \begin{cases} \frac{\rho^n}{n!}P_0 & ;\ n \le c \\ \frac{\rho^n}{c!\,c^{n-c}}P_0 & ;\ n > c \end{cases}$$

The expected number of customers in the system which is waiting and being served can be calculated as follows

$$L_s = L_q + \frac{\lambda}{\mu}$$

wherethe expected number of customers waiting in the queue or queue length is

$$L_q = \left[\frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda\mu}{(c\mu-\lambda)^2}\right]P_0$$

The expected waiting time for a customer in the queue is

$$W_q = \frac{L_q}{\lambda}$$

The expected waiting time for a customer in the system waiting and being served is

$$W_s = W_q + \frac{1}{\mu}$$

## 5. RESULTS AND ANALYSIS

The data for customer arrival, service, and departure time at a popular fast-food restaurant in Pahang during lunch hours 12.00pm to 2.00pm were collected. Based on these sets of data, the Poisson and exponential distributions for each data set are tested and we confirm that all the data sets follow Poisson and exponential probability distributions. The queuing model that best illustrates the operation is 2**M/M/1* which is a multi-channel queuing theory model with 2 service channels. The first 2 *"M"* denotes that the arrival rate and service time are Markovian or memoryless.

Next, we measure the queues' performances by comparing between proposed and current queue systems based on the number of queuesfor each system design. The difference is presented in Figure 2 and Figure 3 below

**Figure 2**Current design multi-line, single-server

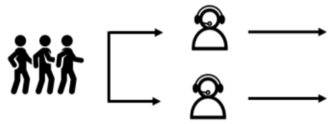

**Figure 3** Proposed design single-line, multi-server

QM for Windows V5 was used to measure the performance of both queuing systems. We determine the efficiency of queuing systems from the calculated value of arrival rate, $\lambda$ and service rate, $\mu$. The result of analysis for each queuing design for the 3-day period of data sets are shown in Table 4 to Table 6 below.

**Table 4** Performance measures on *Day-1*

| Queue Model | $\lambda$ | $\mu$ | Average queue length, $L_q$ (customers) | Average time in queue, $W_q$ (minutes) | Average time in system, $W_s$ (minutes) |
|---|---|---|---|---|---|
| *2*M/M/1* | 22.75 | 43.68 | 0.566 | 1.4931 | 2.8667 |
| | 22.75 | 43.68 | 0.566 | 1.4931 | 2.8667 |
| *M/M/2/∞/FCFS* | 45.50 | 43.68 | 0.3878 | 0.5113 | 1.8850 |

**Table 5** Performance measures on *Day-2*

| Queue Model | $\lambda$ | $\mu$ | Average queue length, $L_q$ (customers) | Average time in queue, $W_q$ (minutes) | Average time in system, $W_s$ (minutes) |
|---|---|---|---|---|---|
| *2*M/M/1* | 25.75 | 49.44 | 0.5661 | 1.3191 | 2.5327 |
| | 25.75 | 49.44 | 0.5661 | 1.3191 | 2.5327 |
| *M/M/2/∞/FCFS* | 51.50 | 49.44 | 0.3878 | 0.4518 | 1.6653 |

**Table 6** Performance measures on *Day-3*

| Queue Model queues | $\lambda$ | $\mu$ | Average queue length, $L_q$ (customers) | Average time in queue, $W_q$ (minutes) | Average time in system, $W_s$ (minutes) |
|---|---|---|---|---|---|
| *2*M/M/1* | 20.75 | 39.84 | 0.5661 | 1.6370 | 3.1430 |
| | 20.75 | 39.84 | 0.5661 | 1.6370 | 3.1430 |
| *M/M/2/∞/FCFS* | 41.50 | 39.84 | 0.3878 | 0.5606 | 2.0666 |

From the tables above, we compare the expected number of waiting customers in the queue ($L_q$) , the expected waiting time of customers in the system ($W_s$), and the expected waiting time of customers in the case of *M/M/1* queues and *M/M/2/∞/FCFS* queue ($W_q$). When each server is analysed with its individual queue, the results are for each server individually. Whereas for the case of one queue, the servers are analysed with one queue for two parallel servers.

From Table 4, the average time for a customer to wait in a queue using the current design in fast-food restaurant (2**M/M/1*) is 1.5 minutes and the average time for a customer to wait in the system in 2.9 minutes. For the proposed system of *M/M/2/∞/FCFS* which is single-line multi-server system design, the average time a customer is in a queue improves to 0.5 minutes and the average time a customer is in a system is 1.9 minutes. By looking at the result from Table 4, the customer can save about 1 minutes in proposed system design compared to the current system design in fast-food restaurant. Furthermore, we can see the average queue length in single queue design is smaller compared to the average queue length in multiple queue design.

From Tables 5 and 6, data from *Day-2* and *Day*-3 have similar results based on the comparison of values of $L_q$, $W_s$, and $W_q$. We can clearly see that the average time of customer in queue and in the system is shorter in single queue design compared to in multiple queue system designs for both sets of data. The average queue length for both set of data is better in the case of the proposed design compared to the current system design. Hence, we can conclude from the expected number of customers and the waiting time of customers in the queue and in the system that the case of one queue *M/M/2/∞/FCFS* is better than two queues 2**M/M/1*.

With respect to the data collected, comparing the average time of customer in queue and in the system and the average queue length for both current design system and proposed design system, we found that the proposed system is better than the current system.

## 7. SUMMARY AND CONCLUSION

The proposed system which is the single-line, multi-server system reduces the waiting time of customers in the queue and the system compared to the existing system which is multi-line, multi-server queue. This implies that a customer needs to wait longer in the queue of existing system compared to in the proposed system. We can say that this study has the evidence that the single queue design will improve the efficiency and the quality of services in this fast-food restaurant. A good queuing design system will give good results on how to increase the restaurant's efficiency.

Referring to the case study, the interpretation from the analysis shows that they should make transformation of their queuing system. We can clearly see that the proposed system has advantage over the existing system design. The customers' waiting time is shortened by applying the proposed system design. As the waiting time of customers is reduced, this will increase the customers' satisfaction. The results from this study should be used to improve customer service and optimize the business' efficiency.

In addition, the data collected during the 3-day period may limit the generalization of our findings because the case study only focused on one fast-food restaurant. Hence, we recommend a further study to cover other fast-food restaurants for a longer period of data collection.

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